Inhomogeneous Einstein–Rosen String Cosmology

Dominic Clancy[1a], Alexander Feinstein[2b], James E. Lidsey[3c] & Reza Tavakol[1d]

[1]*Astronomy Unit, School of Mathematical Sciences,*
Queen Mary & Westfield College, Mile End Road, London, E1 4NS, U.K.

[2]*Dpto. de Física Teórica, Universidad del País Vasco,*
Apdo. 644, E-48080, Bilbao, Spain

[3]*Astronomy Centre and Centre for Theoretical Physics,*
University of Sussex, Brighton, BN1 9QH, U.K.

Abstract

Families of anisotropic and inhomogeneous string cosmologies containing non–trivial dilaton and axion fields are derived by applying the global symmetries of the string effective action to a generalized Einstein–Rosen metric. The models exhibit a two–dimensional group of Abelian isometries. In particular, two classes of exact solutions are found that represent inhomogeneous generalizations of the Bianchi type VI_h cosmology. The asymptotic behaviour of the solutions is investigated and further applications are briefly discussed.

[a]Electronic address: dominic@maths.qmw.ac.uk

[b]Electronic address: wtpfexxa@lg.ehu.es

[c]Electronic address: jel@astr.cpes.susx.ac.uk

[d]Electronic address: reza@maths.qmw.ac.uk

1 Introduction

An important lesson of nonlinear dynamical systems theory is that solutions originating from different regions of state space can (and often do) have qualitatively different modes of behaviour. In this sense, the initial conditions become an important ingredient in determining the dynamical evolution of the system. This then raises the question of how one determines the set of initial conditions that gives rise to a particular dynamical mode of behaviour. The answer to this question requires a full understanding of the underlying dynamics which, in the case of general relativity or string theory, is extremely difficult to establish, given the high degree of nonlinearity and complexity involved in these theories. There is also the added difficulty that despite recent progress in our understanding of M–theory, a definitive non–perturbative formulation of quantum gravity still remains to be developed (see, e.g., Ref. [1] for recent reviews).

This question is particularly relevant in cosmological studies of string or M–theory. The low-energy effective action of the Neveu–Schwarz/Neveu–Schwarz (NS–NS) bosonic sector of string theory contains a multiplet of massless fields $\{g_{\mu\nu}, \phi, B_{\mu\nu}\}$ [2]. The vacuum expectation value of the dilaton, ϕ, determines the string (gravitational) coupling, $g_s^2 \equiv e^\phi$, the gravitational field is determined by the metric, $g_{\mu\nu}$, and $B_{\mu\nu}$ is an antisymmetric, two–form potential. To date, attention has focused on the spatially homogeneous, orthogonal models, where the dilaton field is constant on the surfaces of homogeneity [3, 4, 5]. However, these models apply on scales just below the string scale, and it is precisely this region where spatial inhomogeneities may be important.

A study of inhomogeneous string cosmologies is therefore necessary if further progress is to be made in addressing the question of whether our universe arose out of generic initial conditions. The purpose of the present paper is to derive and study a wide class of 'Einstein–Rosen' string cosmologies with non–trivial dilaton and two–form potential. Einstein–Rosen metrics are interesting for a number of reasons [6]. (For reviews see, e.g., [7, 8]). Spatial homogeneity is broken along one direction and they admit an Abelian isometry group, G_2, that acts on two–dimensional spacelike orbits. They represent a natural generalization of the spatially homogeneous Bianchi cosmologies [9, 10]. Density perturbations in the early universe can be analysed with these backgrounds and the propagation and collision of gravitational plane waves on homogeneous space–times can also be studied within this context [11, 12, 13, 14]. Finally, it has been conjectured that G_2 metrics represent a leading–order approximation to more general solutions near the singularity [15].

There has also been an interesting recent development within string cosmology, namely the *pre–big bang* scenario [16], according to which the rapid increase of the string coupling drives an accelerated, inflationary expansion. The central postulate of this scenario is that the initial state of the universe is in the perturbative regime of small coupling and curvature. This leads to an inflationary phase for sufficiently homogeneous initial conditions [17, 18]. At present, the question of whether in general large spatial inhomogeneities have a significant effect on the naturalness of such initial data is unresolved.

Recently, Barrow and Kunze [19] studied a class of inhomogeneous generalizations of Bianchi I string cosmologies and Feinstein, Lazkoz and Vazquez–Mozo [20] derived an inhomogeneous model by applying duality transformations on the locally rotationally symmetric (LRS) Bianchi type IX cosmology. In this paper we consider the G_2 inhomogeneous general-

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izations of the Bianchi type VI_h universe, where $h < 0$ is the group parameter. This Bianchi model is interesting because it has a non–zero measure in the space of homogeneous initial data and includes the Bianchi type III as a special case ($h = -1$) [21]. Furthermore, the most general spatially homogeneous solutions of the (one–loop) string equations of motion are the Bianchi types III and VI_h, where $h = \{0, -1/2, -2\}$ [19]. It can be shown that these models contain the maximum number of eight free parameters.

We employ non–compact, global symmetries of the field equations to generate inhomogeneous solutions with a non–trivial two–form potential. When the metric admits two commuting spacelike Killing vectors, there exists an infinite–dimensional symmetry on the space of solutions that may be identified infinitesimally with the $\text{O}(2,2)$ current algebra [22, 23]. This symmetry reduces to the Geroch group, corresponding to the $\text{SL}(2,R)$ current algebra, when the dilaton and two–form potential are trivial [24]. The global $\text{SL}(2,R)$ 'S–duality' [25] and $\text{O}(2,2;R)$ 'T–duality' [26] are contained within this symmetry[1]. Application of both these symmetries leads to new, inequivalent solutions.

This paper is organised as follows. In Section 2, we derive inhomogeneous G_2 string cosmologies from a general class of Einstein–Rosen models where the two–form potential is trivial. Two families of solutions representing inhomogeneous generalizations of the Bianchi type VI_h universe are found in Section 3 by directly solving the field equations. The asymptotic behaviour of such models is studied in Section 4. We conclude with a discussion in Section 5.

2 Einstein–Rosen String Cosmology

2.1 String Effective Action

Fundamental strings sweep geodesic surfaces with respect to the string–frame metric, $g_{\mu\nu}$. The four–dimensional, string effective action for the NS–NS fields is given by

$$S = \int d^4x \sqrt{-g}\, e^{-\phi} \left[R + (\nabla\phi)^2 - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right], \tag{2.1}$$

where $H_{\mu\nu\lambda} \equiv \partial_{[\mu} B_{\nu\lambda]}$ is the field strength of the two–form potential. In general, the effective action will also include moduli and vector fields arising from the compactification from higher dimensions. The action is also expected to include a potential term, $V(\phi)$, arising from the non-perturbative sector of the theory, however, the form of such a potential is as yet unknown. These additional terms are neglected in what follows.

In order to take advantage of the highly developed framework of general relativity and its many known exact solutions, it is often more convenient to work in the Einstein frame, where the dilaton field is minimally coupled to gravity. This is achieved by making the conformal transformation

$$\widetilde{g}_{\mu\nu} = e^{-\phi} g_{\mu\nu}. \tag{2.2}$$

Action (2.1) then takes the form

$$S = \int d^4x \sqrt{-\widetilde{g}} \left[\widetilde{R} - \frac{1}{2}(\tilde{\nabla}\phi)^2 - \frac{1}{12} e^{-2\phi} \tilde{H}_{\mu\nu\lambda} \tilde{H}^{\mu\nu\lambda} \right]. \tag{2.3}$$

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[1]We refer to the groups $\text{SL}(2,R)$ and $\text{O}(2,2;R)$ as the S– and T–duality groups, respectively, although at the non–perturbative level the dualities are the discrete subgroups $\text{SL}(2,Z)$ and $\text{O}(2,2;Z)$.

In four dimensions the field strength of the two–form potential is dual to a one–form:

$$\tilde{H}^{\mu\nu\lambda} = \tilde{\epsilon}^{\mu\nu\lambda\kappa} e^{2\phi} \tilde{\nabla}_\kappa \sigma, \tag{2.4}$$

where $\tilde{\epsilon}^{\mu\nu\lambda\kappa}$ is the covariantly constant four–form [25]. The field equations can then be derived from the dual action

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\tilde{R} - \frac{1}{2}\left(\tilde{\nabla}\phi\right)^2 - \frac{1}{2}e^{2\phi}\left(\tilde{\nabla}\sigma\right)^2 \right], \tag{2.5}$$

where σ may be interpreted as a pseudo–scalar 'axion' field.

The generalized Einstein–Rosen G_2 metric is defined in the Einstein frame by the line element [6, 7]

$$ds_e^2 = \tilde{h}_{\alpha\beta}(x^\gamma)dx^\alpha dx^\beta + \tilde{\gamma}_{ab}(x^\gamma)dx^a dx^b, \tag{2.6}$$

where all components are taken to be independent of the spatial coordinates $x^a = (x, y)$. The two commuting, spacelike Killing vectors are $\partial/\partial x$ and $\partial/\partial y$ and $\tilde{h}_{\alpha\beta}$ represents the longitudinal component of the metric. The metric on the surfaces of transitivity is denoted by $\tilde{\gamma}_{ab}$ and the gradient $K_\mu \equiv \partial_\mu (\det \tilde{\gamma}_{ab})^{1/2}$ determines the local behaviour of the model. Solutions represent cylindrical and plane gravitational waves if K_μ is globally spacelike or null, respectively [7, 11]. Cosmological models arise when K_μ is timelike or when the sign of $K_\mu K^\mu$ changes [9].

The longitudinal metric is conformally flat and the line element may therefore be written in the form

$$ds_e^2 = e^f \left(-d\xi^2 + dz^2 \right) + \tilde{\gamma}_{ab} dx^a dx^b, \tag{2.7}$$

where $f = f(\xi, z)$ determines the longitudinal part of the gravitational field. The corresponding line element in the string frame is given by

$$ds_s^2 = e^{\phi+f} \left(-d\xi^2 + dz^2 \right) + \Gamma_{ab} dx^a dx^b, \tag{2.8}$$

where the transverse metric $\Gamma_{ab} \equiv e^\phi \tilde{\gamma}_{ab}$ has determinant $\Gamma \equiv \det\Gamma_{ab}$. We assume throughout this work that all massless fields are independent of the coordinates x^a. Thus, the metric (2.8) also represents a G_2 model.

A considerable simplification occurs in the field equations when the transverse metric is diagonal and separable. In this case, the Einstein and string frame metrics may be written in the form

$$ds_e^2 = e^f(-d\xi^2 + dz^2) + \xi(e^p dx^2 + e^{-p}dy^2) \tag{2.9}$$

and

$$ds_s^2 = e^{f+\phi} \left(-d\xi^2 + dz^2 \right) + \xi e^\phi \left(e^p dx^2 + e^{-p}dy^2 \right), \tag{2.10}$$

respectively, where $p = p(\xi, z)$ represents the transverse part of the gravitational field. In this case, the volume of the transverse space in the string frame is determined by

$$\Gamma = \xi^2 e^{2\phi}. \tag{2.11}$$

In some settings, it proves convenient to define new variables

$$z \equiv e^{-2Z} \cosh(2t)$$
$$\xi \equiv e^{-2Z} \sinh(2t), \tag{2.12}$$

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which transform the metric (2.9) to

$$ds_e^2 = 4e^h(-dt^2 + dZ^2) + e^{-2Z}\sinh(2t)(e^p dx^2 + e^{-p} dy^2), \qquad (2.13)$$

where $h \equiv f - 4Z$.

In the present context, an inhomogeneous string cosmology is parametrized by the massless degrees of freedom $\{g_{\mu\nu}, \phi, B_{\mu\nu}\}$. A vacuum solution to Einstein gravity may then be represented by $\{g_{\mu\nu}, 0, 0\}$ and a solution with a trivial two–form potential by $\{g_{\mu\nu}, \phi, 0\}$. We refer to this latter class of solution as 'dilaton–vacuum' solutions. In general, the one–loop field equations of motion for G_2 backgrounds derived from the actions (2.1) and (2.5) are difficult to solve. In view of this, we employ the non–compact, global symmetries that arise when the metric admits two Abelian isometries to generate a wide class of inhomogeneous string cosmologies with a non–trivial two–form potential from dilaton–vacuum solutions.

2.2   O(2,2) Symmetry

The global $O(2,2)$ symmetry applies when there exist two Abelian isometries and the only non–trivial component of the two–form potential is $B_{xy} = B_{xy}(\xi, z)$ [26]. This symmetry is manifest in the string frame and generates fractional linear transformations on the two–form potential and the components of the transverse metric, Γ_{ab}. When the transverse metric is non–diagonal, the four degrees of freedom $\{B_{xy}, \Gamma_{ab}\}$ parametrize the $O(2,2)/[O(2) \times O(2)]$ coset [26]. The isomorphism $O(2,2) = SL(2,R) \times SL(2,R)$ then implies that these may be arranged in terms of two complex coordinates [27]

$$\tau \equiv \frac{\Gamma_{xy}}{\Gamma_{yy}} + i\frac{\sqrt{\Gamma}}{\Gamma_{yy}} \qquad (2.14)$$

$$\rho \equiv B_{xy} + i\sqrt{\Gamma}. \qquad (2.15)$$

Suppose the metric (2.10) represents a vacuum or dilaton–vacuum solution for some appropriate form of the dilaton, $\phi = \phi(\xi, z)$. An $O(2,2)$ transformation is then generated in terms of the two $SL(2,R)$ transformations:

$$\bar{\rho} = \frac{a\rho + b}{c\rho + d}, \qquad \bar{\tau} = \tau \qquad (2.16)$$

$$\bar{\tau} = \frac{a'\tau + b'}{c'\tau + d'}, \qquad \bar{\rho} = \rho, \qquad (2.17)$$

where $ad - bc = 1$ and $a'd' - b'c' = 1$. Under a general $O(2,2)$ transformation, the dilaton transforms to

$$e^{\bar{\phi}} = e^{\phi}\left(\frac{\bar{\Gamma}}{\Gamma}\right)^{1/2} \qquad (2.18)$$

and the longitudinal part of the string frame metric remains invariant, i.e.,

$$\bar{f} = f + \phi - \bar{\phi}. \qquad (2.19)$$

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Since the transformations (2.17) leave the two–form potential invariant, (2.16) must be employed to generate such a field from a dilaton–vacuum seed solution. Applying (2.16) implies that

$$\bar{\Gamma} = \frac{\Gamma}{(d^2 + c^2\Gamma)^2}, \tag{2.20}$$

$$\bar{B}_{xy} = \frac{ac\Gamma + bd}{c^2\Gamma + d^2}, \tag{2.21}$$

$$e^{\bar{\phi}} = \frac{e^\phi}{d^2 + c^2\Gamma}, \tag{2.22}$$

where the last transformation follows from (2.11) and (2.20). The dual metrics in the string and Einstein frames are then given by

$$d\bar{s}_s^2 = e^{f+\phi}\left(-d\xi^2 + dz^2\right) + \frac{\xi e^\phi}{d^2 + c^2\xi^2 e^{2\phi}}\left(e^p dx^2 + e^{-p} dy^2\right) \tag{2.23}$$

and

$$d\bar{s}_e^2 = e^f\left(d^2 + c^2\xi^2 e^{2\phi}\right)\left(-d\xi^2 + dz^2\right) + \xi\left(e^p dx^2 + e^{-p} dy^2\right), \tag{2.24}$$

respectively. We remark that the transverse metric in the Einstein frame, $\tilde{\gamma}_{ab}$, is invariant under the transformations (2.16). When $d = 0$ and $c = 1$, the volume of the transverse space in the string frame, as given by Eq. (2.20), is inverted. The transformations (2.16) therefore represent a 'T–duality'.

2.3 SL(2,R) Symmetry

The global SL$(2, R)$ symmetry of the string effective action (2.1) becomes manifest in the Einstein frame. The action (2.5) may be written as a non–linear sigma–model, where the dilaton and axion fields parametrize the SL$(2, R)$/U(1) coset [25]. The effective action is therefore invariant under global SL$(2, R)$ transformations. These act non–linearly on the complex scalar field $\chi \equiv \sigma + ie^{-\phi}$ such that the transformed field is given by

$$\bar{\chi} = \frac{A\chi + B}{C\chi + D}, \tag{2.25}$$

where $\{A, B, C, D\}$ are real numbers satisfying $AD - BC = 1$. The Einstein frame metric transforms as a singlet under this SL$(2, R)$ transformation and the dual string frame metric is therefore given by

$$d\bar{s}_s^2 = e^{\bar{\phi}-\phi} ds_s^2. \tag{2.26}$$

For the special case where $C^2 = 1$ and $\sigma = -D/C$, the SL$(2, R)$ transformation (2.25) yields $\bar{\phi} = -\phi$, which corresponds to an inversion of the string coupling $\bar{g}_s = g_s^{-1}$. The transformations (2.25) therefore represent a strong/weak–coupling 'S–duality'.

Starting with $\sigma = 0$, a solution with non–trivial axion field may be generated directly from a given dilaton–vacuum solution of the generic form (2.9) by applying Eq. (2.25). The dual solutions are given by

$$d\bar{s}_s^2 = e^{\bar{\phi}+f}\left(-d\xi^2 + dz^2\right) + \xi e^{\bar{\phi}}\left(e^p dx^2 + e^{-p} dy^2\right) \tag{2.27}$$

$$e^{\bar{\phi}} = C^2 e^{-\phi} + D^2 e^{\phi} \tag{2.28}$$

$$\bar{\sigma} = \frac{ACe^{-\phi} + BDe^{\phi}}{C^2 e^{-\phi} + D^2 e^{\phi}}. \tag{2.29}$$

To summarize thus far, we have seen how two inequivalent classes of inhomogeneous string cosmologies with a non–trivial two–form potential can be derived from a given dilaton–vacuum solution, by employing the non–compact, global $\mathrm{SL}(2, R)$ and $\mathrm{O}(2, 2)$ symmetries of the model. The dual solutions may be referred to as 'dilaton–axion' cosmologies. In all cases, they are parametrized in terms of the functions $\{f, p, \phi\}$ that define the seed dilaton–vacuum solutions.

Thus, the asymptotic behaviour of these models can be investigated directly once the seed solution has been specified. We therefore proceed in the following Section to derive two classes of inhomogeneous dilaton–vacuum cosmologies that may be viewed as generalizations of the homogeneous Bianchi type VI_h universe.

3 Inhomogeneous Dilaton–Vacuum Cosmology

3.1 Cosmological Field Equations

When the two–form potential vanishes, action (2.5) reduces to that for a massless, minimally coupled scalar field. For the metric (2.9) the field equations then take the form [28]

$$\dot{f} = -\frac{1}{2\xi} + \frac{\xi}{2} \left(\dot{p}^2 + p'^2 + \dot{\phi}^2 + \phi'^2 \right) \tag{3.1}$$

$$f' = \xi \left(\dot{p}p' + \dot{\phi}\phi' \right), \tag{3.2}$$

$$\ddot{p} + \frac{1}{\xi}\dot{p} - p'' = 0, \tag{3.3}$$

$$\ddot{\phi} + \frac{1}{\xi}\dot{\phi} - \phi'' = 0, \tag{3.4}$$

in which overdots (primes) denote differentiation with respect to the timelike variable ξ (spacelike variable z). Eqs. (3.3) and (3.4) are the integrability conditions for the system (3.1) and (3.2). The field equations (3.1)–(3.4) are invariant under the simultaneous interchange $p \leftrightarrow \phi$. Indeed, the wave equations (3.3) and (3.4) are formally equivalent to the cylindrically symmetric wave equation in flat space, for which the general solution is known. A number of cosmologically relevant solutions satisfying different boundary conditions have been considered previously [7, 8, 29]. An important feature of these equations is that they are linear, which implies that new solutions may be constructed from superpositions of known solutions.

Here we consider a linear superposition of solutions of the form:

$$p = k \ln(\xi) - m \cosh^{-1}\left(\frac{z}{\xi}\right) + \epsilon_1 \int_0^\infty [c_1(z)J_0(l\xi) + c_2(z)N_0(l\xi)] \, dl \tag{3.5}$$

$$\phi = \alpha \ln(\xi) - \beta \cosh^{-1}\left(\frac{z}{\xi}\right) + \epsilon_2 \int_0^\infty [c_3(z)J_0(l\xi) + c_4(z)N_0(l\xi)] \, dl, \tag{3.6}$$

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where $\{\alpha, \beta, k, m\}$ and $\{\epsilon_1, \epsilon_2\}$ are constants, J_0 and N_0 are zero–order Bessel functions of the first and second kind and the coefficients $c_i = c_i(z)$ are defined by

$$
\begin{align}
c_1(z) &= C_l \cos(lz) + D_l \sin(lz) \tag{3.7} \\
c_2(z) &= F_l \cos(lz) + G_l \sin(lz) \tag{3.8} \\
c_3(z) &= H_l \cos(lz) + L_l \sin(lz) \tag{3.9} \\
c_4(z) &= U_l \cos(lz) + V_l \sin(lz), \tag{3.10}
\end{align}
$$

where C_l, D_l, etc., are arbitrary constants for each value of l. In both Eqs. (3.5) and (3.6), the last term represents the most general *separable* solution to Eqs. (3.3) and (3.4). The remaining two terms represent other, in general inhomogeneous, solutions[2]. The importance of these other solutions is that they include a number of spatially homogeneous Bianchi models as special cases. The Bianchi models admit a three–dimensional group of isometries, G_3, that acts simply transitively on three–dimensional spacelike orbits [10]. The G_3 contains an Abelian subgroup G_2 for the types I–VII$_h$ and the LRS types VIII and IX [9, 30]. Certain G_2 models may therefore be viewed as inhomogeneous generalizations of these Bianchi cosmologies. In particular, for solutions of the form (3.5)–(3.10), one has the following sub–classes of solutions:

1. The class with $\epsilon_1 = \epsilon_2 = m = \beta = 0$ corresponds to the homogeneous orthogonal Bianchi type I models containing a stiff perfect fluid [31]. They reduce to the Kasner solution in the vacuum limit ($\alpha = 0$) [32].

2. The class with $m = \beta = 0$, $\epsilon_1 = \epsilon_2 = 1$ corresponds to an inhomogeneous generalisation of the Bianchi I models. Charach and Malin first considered solutions similar to these by imposing a three–torus topology on the spatial sections [28]. This effectively converts the integral over l into an infinite sum, which also has the consequence of simplifying the solution of the remaining field equations for f. Adams *et al.* [12] further considered the vacuum case ($\phi = 0$).

3. The class with $\epsilon_1 = \epsilon_2 = 0$, subject to the constraint $\beta^2 - \alpha^2 + m^2 - k^2 - 3 = 0$, where $k \equiv (-h)^{-1/2}$. In general, these models represent tilted stiff perfect fluid Bianchi type VI$_h$ cosmologies [33], since the fluid velocity vector associated with the dilaton field is not orthogonal to the group orbits (surfaces of homogeneity). They reduce to the Bianchi type III and V models when $k^2 = 1$ and $k = 0$, respectively [34]. In the vacuum limit ($\alpha = \beta = 0$), the solution reduces to the Ellis–MacCallum type VI$_h$ cosmology [35].

4. The class with $\epsilon_1 = \epsilon_2 = 0$ corresponds to the inhomogeneous generalisations of the Bianchi type III, V and VI$_h$ models first considered by Wainwright *et al.* [33]. In general, however, these models suffer from spacelike singularities and their status as physical cosmologies is uncertain [7].

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[2]The first term is also present in the third term and is therefore not a different solution. It is separated out for later consideration.

An alternative way of considering inhomogeneous generalisations of the Bianchi type III, V and VI$_h$ models is to specify $\epsilon_1 = \epsilon_2 = 1$ and impose the constraint

$$\beta^2 - \alpha^2 + m^2 - k^2 - 3 = 0. \tag{3.11}$$

Vacuum solutions of this type were considered by Adams et al. [13], who concluded that the inhomogeneous structure of the initial cosmic singularity could evolve into gravitational waves propagating over a homogeneous background at late times. In view of the ambiguities associated with interpreting the Wainwright *et al.* [33] solutions in a cosmological context, our primary interest in the present paper is in this new class of models. We also remark that a subset of inhomogeneous Bianchi I models (item 2) is also included within this class since these latter models correspond to the particular solution $\beta = m = 0$.

The remaining field equations (3.1) and (3.2) for the longitudinal metric function f may now be solved in principle by substituting in the derivatives of Eqs. (3.5) and (3.6). However, solving Eq. (3.1) is non–trivial, because the right-hand-side contains integrals over l that originate from the integral wave-train terms of Eqs. (3.5) and (3.6). Unfortunately, these can not be expressed in a closed form and some simplifying assumptions must therefore be made in order to proceed analytically.

In view of this, we consider two separate schemes. In the first, we restrict the analysis to a single mode, l, which could be viewed as dominating over all the other modes. The choice of one mode allows the integrals over l to be dropped[3]. In the second case, we assume the amplitudes of each of the modes in Eqs. (3.5) and (3.6) are equal, i.e., we specify $C_l = C$, $D_l = D$, etc.

We first integrated Eq. (3.1) to deduce an expression for f containing an unknown function of integration $f_1(z)$. In each of the cases we considered, it was found that Eq. (3.2) was then trivially satisfied for constant $f_1(z)$. We now proceed to present the two classes of solutions, together with their homogeneous limits.

3.2   Homogeneous Solutions

The homogeneous limit of these solutions is determined by specifying $\epsilon_1 = \epsilon_2 = 0$ in Eqs. (3.5) and (3.6) [33]. Eq. (3.1) can then be integrated to yield the longitudinal component of the gravitational field in the metric (2.9):

$$
\begin{aligned}
f_{hom} &= C_1 + \frac{1}{2}\left(\alpha^2 + \beta^2 + k^2 + m^2 - 1\right)\ln(\xi) - \frac{1}{2}\left(\beta^2 + m^2\right)\ln(z^2 - \xi^2) \\
&\quad - (\alpha\beta + km)\cosh^{-1}\left(\frac{z}{\xi}\right),
\end{aligned}
\tag{3.12}
$$

where C_1 is an arbitrary constant of integration and the constraint equation (3.11) applies. In terms of the variables (2.12) this component is given by

$$h_{hom} = C_1 + \frac{1}{2}\left(\alpha^2 + \beta^2 + k^2 + m^2 - 1\right)\ln\sinh(2t) + (km + \alpha\beta)\ln\tanh t \tag{3.13}$$

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[3]Particular solutions of the field equations that consist of superpositions of two or more modes may also be found by employing a discrete summation over l in place of the integral in Eqs. (3.5) and (3.6).

in Eq. (2.13). The transverse component of the metric and the dilaton field respectively take the following forms

$$p_{hom} = -2kZ + k \ln \sinh(2t) + m \ln \tanh t \qquad (3.14)$$

$$\phi_{hom} = -2\alpha Z + \alpha \ln \sinh(2t) + \beta \ln \tanh t. \qquad (3.15)$$

The hypersurfaces $t = $ constant represent the surfaces of homogeneity. Since for $\alpha \neq 0$, the dilaton field depends on the spatial variable, Z, the fluid velocity as measured by $u_\mu = \phi_{,\mu}/(-\phi_{,\nu}\phi^{,\nu})^{1/2}$ is not orthogonal to the group orbits. Thus, this solution may be interpreted as a tilted stiff fluid type VI_h solution.

3.3 Single-mode solutions

After restricting the analysis to a single mode, l, the integral of Eq. (3.1) is readily found. The resulting solution may be expressed in terms of its homogeneous, gravitational wave and scalar wave components:

$$f = f_{hom} + f_{gw} + f_{sw}, \qquad (3.16)$$

where the homogeneous component, f_{hom}, is given by Eq. (3.12), the gravitational wave component is given by

$$
\begin{aligned}
f_{gw} = {}&+\frac{\xi^2}{4}\left([c_1' J_0(l\xi) + c_2' N_0(l\xi)]^2 + [c_1' J_1(l\xi) + c_2' N_1(l\xi)]^2\right) \\
&+\frac{l^2}{4}\xi^2\left([c_1 J_0(l\xi) + c_2 N_0(l\xi)]^2 + [c_1 J_1(l\xi) + c_2 N_1(l\xi)]^2\right) \\
&-\frac{l}{2}\xi\left[c_1 J_0(l\xi) + c_2 N_0(l\xi)\right]\left[c_1 J_1(l\xi) + c_2 N_1(l\xi)\right] + k\left(c_1 J_0(l\xi) + c_2 N_0(l\xi)\right) \\
&-mlzc_1 \int \frac{J_1(l\xi)}{\sqrt{z^2 - \xi^2}}\,d\xi - mlzc_2 \int \frac{N_1(l\xi)}{\sqrt{z^2 - \xi^2}}\,d\xi \\
&-mc_1' \int \frac{\xi J_0(l\xi)}{\sqrt{z^2 - \xi^2}}\,d\xi - mc_2' \int \frac{\xi N_0(l\xi)}{\sqrt{z^2 - \xi^2}}\,d\xi
\end{aligned} \qquad (3.17)
$$

and the scalar-wave component f_{sw} is determined by making the substitutions

$$k \to \alpha, \quad m \to \beta, \quad c_1(z) \to c_3(z), \quad c_2(z) \to c_4(z) \qquad (3.18)$$

in Eq. (3.17). The gravitational and scalar wave components encode all the inhomogeneous contributions to the longitudinal component of the gravitational field. The integrals in the expressions for f_{gw} and f_{sw} can actually be performed, provided the integrand is first expressed as a series. The result is a series which reduces to a closed form expression in the asymptotic late-time limit.

3.4 Equal-amplitude solutions

When all the modes in the integral wave–trains of Eqs. (3.5) and (3.6) have equal weighting, we may employ the identities

$$\int_0^\infty J_0(l\xi)\,f(lz)\,dl = \begin{cases} \frac{1}{\sqrt{z^2-\xi^2}}, & \text{if} \quad f(lz) = \sin(lz) \\ 0, & \text{if} \quad f(lz) = \cos(lz) \end{cases} \qquad (3.19)$$

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$$\int_0^\infty N_0(l\xi)\, f(lz)\, dl = \begin{cases} \frac{2}{\pi} \frac{\cosh^{-1}(z/\xi)}{\sqrt{z^2 - \xi^2}}, & \text{if} \quad f(lz) = \sin(lz) \\ \frac{-1}{\sqrt{z^2 - \xi^2}}, & \text{if} \quad f(lz) = \cos(lz) \end{cases} \qquad (3.20)$$

in order to integrate Eq. (3.1). To avoid unnecessary complications, we further restrict our attention to the class of solutions where the contributions from the Neumann functions vanish $(c_2 = c_4 = 0)$[4]. The transverse component of the gravitational field and the dilaton are then given by

$$p = k \ln(\xi) - m \cosh^{-1}\left(\frac{z}{\xi}\right) + \frac{F}{\sqrt{z^2 - \xi^2}} \qquad (3.21)$$

$$\phi = \alpha \ln(\xi) - \beta \cosh^{-1}\left(\frac{z}{\xi}\right) + \frac{M}{\sqrt{z^2 - \xi^2}}, \qquad (3.22)$$

respectively, where F and M are arbitrary constants. The longitudinal component of the gravitational field, f, may now be determined by substituting Eqs. (3.21) and (3.22) into Eqs. (3.1) and (3.2) and integrating. One finds that

$$f = f_{hom} + \frac{F^2 + M^2}{4(z^2 - \xi^2)^2}\xi^2 + \frac{mF + \beta M}{z^2 - \xi^2} z + \frac{kF + \alpha M}{\sqrt{z^2 - \xi^2}}. \qquad (3.23)$$

In terms of the variables defined in Eq. (2.12), it is given by

$$h = h_{hom} + (kF + \alpha M)e^{2Z} + (mF + \beta M)e^{2Z}\cosh(2t) + \frac{1}{4}(F^2 + M^2)e^{4Z}\sinh^2(2t), \quad (3.24)$$

where h_{hom} is given by Eq. (3.13). The transverse component and dilaton are given by

$$p = -2kZ + k \ln \sinh(2t) + m \ln \tanh t + Fe^{2Z} \qquad (3.25)$$

$$\phi = -2\alpha Z + \alpha \ln \sinh(2t) + \beta \ln \tanh t + Me^{2Z} \qquad (3.26)$$

when expressed in terms of these variables.

4 Asymptotic Behaviour

In this section we consider some aspects of the asymptotic behaviour of the inhomogeneous string cosmologies derived above. The axion field in the dual solution (2.27)–(2.29) tends to a constant value in the limits $\phi \to \pm\infty$. It is therefore dynamically negligible in these limits. Eq. (2.28) implies that for all solutions generated by the SL(2, R) transformation (2.25), there exists a lower bound on the value of the dilaton field if C and D are non–zero. This implies the existence of a *lower* (non-vanishing) bound on the string coupling which, in the context of M–theory, in turn implies the existence of a lower bound on the radius of the eleventh dimension[5], $\mathcal{R}_{11}$, since $\mathcal{R}_{11} \propto e^{\phi/3}$ [1, 36].

In the class of dual solutions (2.23), it follows from Eq. (2.21) that the two–form potential tends to a constant in the limits where $\Gamma \to 0$ and $\Gamma \to \infty$, i.e., when the volume of the transverse space measured in the string frame becomes vanishingly small or arbitrarily

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[4]The solution may also be found for the case when the Neumann functions are included.

[5]We are assuming implicitly that the extra six spatial dimensions are fixed.

large. Thus, the two–form potential effectively decouples from the field equations in these limits. The limiting behaviour of the dilaton field in the dual solution follows from Eq. (2.22). In particular, when $\Gamma \to 0$ and $d \neq 0$, the dilaton asymptotically tends to the form it has in the seed solution (up to a constant linear shift). The same conclusion applies for $\bar{\Gamma}$. In this limit, therefore, the dual cosmology asymptotes to the original seed, dilaton–vacuum solution. In the opposite limit, where Γ diverges, the dual cosmology asymptotes to the solution that is derived by specifying $d = 0$ in Eq. (2.16).

The above discussion applies to any class of string cosmologies derived with the symmetry transformations discussed in Section 2. To illustrate this further, we consider the asymptotic behaviour of the string cosmologies (2.23) derived from the equal–amplitude dilaton–vacuum, seed solution (3.21)–(3.23).

We recall that in determining the asymptotic behaviour of cosmological models the choice of time gauge is important. Since the models considered here may be viewed as describing inhomogeneous waves propagating over homogeneous Bianchi backgrounds, a reasonable measure of early and late times is provided by t in the coordinate chart $\{t, Z, x, y\}$. As we shall see, the asymptotic behaviour in the limits of small and large t corresponds to either $\Gamma \to 0$ or ∞ in the solution (2.23). This implies that the class of string cosmologies (2.23) asymptotes between two dilaton–vacuum solutions, where the two–form is dynamically negligible. In effect, this field induces the transition between the two dilaton–vacuum limits. A similar conclusion holds for the dual solution (2.27).

In the limit, $t \to +\infty$, the relevant terms in the asymptotic forms of Eqs. (3.21), (3.22) and (3.24) are

$$p \approx 2k(t - Z) + F\,e^{2Z} \tag{4.1}$$

$$\phi \approx 2\alpha(t - Z) + M\,e^{2Z} \tag{4.2}$$

$$h \approx (\alpha^2 + \beta^2 + k^2 + m^2 - 1)t + \frac{1}{16}(F^2 + M^2)e^{4(Z+t)}. \tag{4.3}$$

In terms of the coordinate pair $\{\xi, z\}$, this implies that

$$p \approx k\ln(\xi) + \frac{F}{\sqrt{z^2 - \xi^2}} \tag{4.4}$$

$$\phi \approx \alpha\ln(\xi) + \frac{M}{\sqrt{z^2 - \xi^2}} \tag{4.5}$$

$$f \approx \frac{1}{2}(\alpha^2 + \beta^2 + k^2 + m^2 - 1)\ln(\xi) + \frac{(F^2 + M^2)\xi^2}{4(z^2 - \xi^2)^2}. \tag{4.6}$$

We note that, in view of Eq. (2.12), the singularity at $z = \xi$ only corresponds to $(\xi, z) \to (\infty, \infty)$ and therefore cannot be reached in a finite t-time along *any* curve.

In the limit $t \to 0$, the corresponding limiting forms of the metric components and dilaton field are

$$p \approx -2kZ + (k + m)\ln t + Fe^{2Z} \tag{4.7}$$

$$\phi \approx -2\alpha Z + (\alpha + \beta)\ln t + Me^{2Z} \tag{4.8}$$

$$h \approx \frac{1}{2}\left[\alpha^2 + \beta^2 + k^2 + m^2 + 2(km + \alpha\beta) - 1\right]\ln(t) + [(k + m)F + (\alpha + \beta)M]\,e^{2Z}. \tag{4.9}$$

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The limits of the determinant Γ are deduced by substituting Eq. (2.12) and Eq. (4.2) or (4.8) into Eq. (2.11):

$$\Gamma(t \to +\infty) \approx \exp\left[4(\alpha+1)(t-Z) + 2Me^{2Z}\right] \tag{4.10}$$

$$\Gamma(t \to 0) \approx 4\exp\left[-4(1+\alpha)Z + 2Me^{2Z} + 2(1+\alpha+\beta)\ln t\right]. \tag{4.11}$$

The behaviour of the dilaton field in the limit where the determinant diverges is most readily determined by substituting Eq. (2.11) into Eq. (2.22). We find that $\bar{\phi} \approx -\phi - 2\ln\xi - 2\ln c$. This is interesting, because for finite Z, Eq. (4.2) implies that $\phi \propto \alpha\ln\xi$ for $t \to \infty$. For $\alpha \gg 1$, therefore, $\bar{\phi} \approx -\phi$ and, in this sense, the strongly coupled limit of the dual solution may be viewed as the weakly coupled limit of the seed solution, and vice–versa.

In taking the early or late time limits of an inhomogeneous cosmological model, one has to take account of both direction and time. Here, we have inhomogeneity in the Z-direction only. The question that then arises is whether the Z-dependent terms in the above expressions dominate over the t–dependent ones. For any finite Z, the time–dependent terms eventually dominate as $t \to \{0, \infty\}$. A possible ambiguity in the limit arises, however, if we allow $Z(t) \to \infty$ sufficiently fast relative to the t–dependent terms.

It is helpful to consider the simple set of straight lines $\mathcal{V} = \{Z = \rho t + \kappa \mid \rho, \kappa \in \mathcal{R}\}$, as probes with which to study the asymptotic behaviour[6]. In this case, it follows from Eq. (4.10) that

$$\Gamma(t \to \infty) \approx \begin{cases} \exp[4(1+\alpha)(t-Z)] & \text{if} \quad \rho \leq 0 \\ \exp[2Me^{2Z}] & \text{if} \quad \rho > 0 \end{cases}. \tag{4.12}$$

The sign of ρ determines the term that dominates in Eq. (4.12), with $\rho > 0$ ($\rho < 0$) corresponding to motion in the positive (negative) Z–direction while $\rho = 0$ corresponds to moving along $Z = $ constant trajectories. Thus, for $\rho \leq 0$, $\Gamma \to \infty$ for $\alpha > -1$, while $\Gamma \to \infty$ for $M > 0$ and $\rho > 0$. On the other hand, the determinant becomes vanishingly small if $\alpha < -1$ and $\rho \leq 0$ or if $M < 0$ and $\rho > 0$.

In the early time limit, $Z \to \kappa = $ constant for the paths we are considering. Eq. (4.11) then implies that $\Gamma \propto t^{2(1+\alpha+\beta)}$. It follows that $\Gamma \to 0$ if $\alpha + \beta > -1$ and $\Gamma \to \infty$ if $\alpha + \beta < -1$. The limiting forms of the dilaton and the transverse and longitudinal components of the metric are determined by allowing $Z \to \kappa$ in Eqs. (4.7)–(4.9). Transforming to the synchronous frame by defining

$$\tau \equiv \int^t dt' e^{h(t')/2} \tag{4.13}$$

then implies that, for both seed and dual solutions, the G_2 line-element (2.6) in either the Einstein or string frames qualitatively takes the form

$$ds^2 = -d\tau^2 + A_1(\kappa)\tau^{a_1}dx^2 + A_2(\kappa)\tau^{a_2}dy^2 + A_3(\kappa)\tau^{a_3}dz^2, \tag{4.14}$$

where the constants a_i can be expressed in terms of $\{k, m, \alpha, \beta\}$ and A_i depend on κ. This solution represents an inhomogeneous generalization of the Kasner-Belinskii-Khalatnikov (KBK) solution [38], in the sense that at each Z, parametrised here by the constant κ, the universe describes a particular KBK solution.

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[6]The precise form of these curves is not important, their utility derives from the fact that they may be employed to probe the three dynamically important cases $Z(t) \to \pm\infty$, $Z(t) = $ finite.

Thus, the determinant of the transverse space in the string frame vanishes or diverges in the early and late time limits. The precise behaviour depends on the constants that arise in the seed dilaton–vacuum solution. These constants are arbitrary modulo the constraint equation (3.11). In both limits, the two–form potential asymptotes to a constant value and the dilaton field tends to its original form when $\Gamma \to 0$.

5   Conclusion and Discussion

In this paper we have employed the global symmetries of the string effective action to derive two families of inhomogeneous string cosmologies from a generalized Einstein–Rosen metric admitting an Abelian group of isometries, G_2. The solutions were parametrized in terms of the metric functions of the seed solution. Thus, the qualitative behaviour of these models can be determined directly from the asymptotic form of the original metric. Inhomogeneous generalizations of the Bianchi type VI_h string cosmologies containing a non–trivial dilaton and two–form potential were derived. In general, these fields induce inhomogeneities that may be viewed as scalar and gravitational waves propagating over a homogeneous background.

One of the main applications of Einstein-Rosen models arises because many of the non–perturbatively *exact* superstring backgrounds constructed from the gauged Wess–Zumino–Witten (WZW) models admit two abelian isometries[7]. Hence, given our current knowledge of string cosmology and conformal field theory, the G_2 Einstein–Rosen cosmologies derived within the context of the low energy effective action represent a set of models that is closely related to many exact string solutions in four dimensions. In this sense, it is of primary importance to understand the dynamical behaviour of these solutions. In particular, one question that arises is whether the G_2 solutions to the low energy string equations of motion discussed in this work asymptote towards known WZW models in the early– or late–time limits.

Since our solutions are valid in the weak coupling regime, they should correctly represent the asymptotic states of perturbatively exact string cosmological models at future, or past, timelike infinity. On the other hand, in the strong coupling regime, i.e. near the big-bang singularity, one expects that higher–order corrections to the perturbative theory, as well as non–perturbative string effects, should become increasingly important. Hence, in this regime the qualitative behaviour of these solutions may deviate somewhat from that of solutions derived from the full M/string theory. However, there are reasons to believe that G_2 solutions should nevertheless provide a generic description of cosmological models in the vicinity of a singularity. A major incentive for this comes from the long standing conjecture of Belinski and Khalatnikov [38, 39, 40]. This states that on the approach to the cosmological singularity, the generalized Einstein-Rosen metrics may play the role of the leading–order approximation to the *general* solution of conventional Einstein gravity. It is therefore important to study the behaviour of these models in the $t \to 0$ limit as well.

In addition such solutions provide a useful framework within which to study a number of other topics in early universe string cosmology. In particular, they serve as a theoretical setting for investigating the pre–big bang inflationary scenario [16]. In the context of this

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[7]See, for instance, Ref. [37] for a review of the gauged WZW models.

scenario, one is usually interested in solutions where both the curvature and effective string coupling (the dilaton) diverge as $t \to 0$. So, from Eq. (4.8), it follows that one needs $\alpha + \beta < 0$ in order to satisfy the requirements for pre–big bang inflation in the string cosmologies generated from the equal–amplitude, dilaton–vacuum solutions. These solutions possess a Kasner-like early time limit which under time reversal makes them compatible with pre–big bang inflation. However, in order for the inflation to end, this scenario requires a mechanism for inducing a graceful exit into the standard, post–big bang phase. At present, this problem is unresolved and it is possible that the inhomogeneous singularity may pose a further complication in addressing the exit problem. It would be interesting to consider the homogenization of the universe within the context of these solutions.

The initial conditions for the pre–big bang scenario are based on the assumption that both the curvature and coupling are sufficiently small enough to ensure that the universe is in the perturbative regime. An important question in connection with this scenario is the naturalness of these initial conditions [41]. An attempt has recently been made to address this question by conjecturing that the past attractor of the inflationary solution is likely to be that of the Milne universe [17]. If true, this would go some way towards establishing the 'naturalness' of the scenario. More general initial states were also recently considered [18, 42]. The solutions discussed above possess a range of possible late time limits, which in general are not Milne–like. This suggests that this class of models is not generally compatible with the conjecture of Ref. [17].

Furthermore, there exists a lower bound on the string coupling for models generated by the $\mathrm{SL}(2, R)$ transformation (2.25) and this has implications for the range of initial values that such a parameter can take. This in turn leads to an upper limit on the amount of inflation that can occur before higher–order effects become significant [41].

The solution generating techniques discussed in Section 2 can be incorporated into more general algorithms. Although the models presented in this paper break spatial homogeneity along one direction, they still exhibit a certain degree of symmetry and it is important to develop further techniques that lead to more general solutions. Recently, by extending a previous method [43], an algorithm was presented that generates inhomogeneous G_1 scalar field cosmologies exhibiting a single isometry from matter filled and vacuum G_2 backgrounds [44]. Such models break homogeneity in two spatial directions. The discussion of Ref. [44] was placed within the context of Einstein gravity and it would be interesting to adapt this algorithm to string cosmology. In principle, a family of inhomogeneous G_1 string cosmologies could then be generated from the G_2 solutions discussed in Section 2.

The string effective action exhibits a further discrete symmetry when there exits a G_2 isometry [22]. This 'mirror' symmetry interchanges the transverse metric degrees of freedom with the dilaton and axion fields and leads to a new solution with a different spacetime interpretation [22, 45]. When the axion field is trivial, the symmetry reduces to the simultaneous interchange $p \leftrightarrow \phi$ in Eqs. (3.1)–(3.4). In particular, the negatively curved Friedman–Robertson–Walker string cosmology [4] may be generated in this way from a vacuum Bianchi type V model [45]. A third class of inhomogeneous string solutions with a non–trivial two–form potential may therefore be found by applying this discrete symmetry to the G_2 backgrounds derived in this work.

The G_2 backgrounds we have discussed are parametrized by non–trivial fields from the NS–NS sector of the string effective action. This sector is common to all five perturbative

string effective actions and the solutions may therefore be viewed as truncated solutions of both the heterotic and type II theories. The type IIB theory also has a non–trivial Ramond–Ramond (RR) sector, consisting of an additional axion field and a two–form potential [2]. These fields differ from those of the NS–NS sector in that they do not couple directly to the dilaton field in the effective action [36, 46]. There are further symmetry transformations that can be applied to generate a non–trivial RR sector from a given NS–NS background [47]. Our solutions therefore represent seeds for investigating the role of RR fields in inhomogeneous string cosmologies. To date, such fields have only been studied in an homogeneous setting.

Acknowledgments

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